

## Solving Trig Equations Using Advanced Identities HW (3.10 + 3.12)

For each question that involves applying a trigonometric identity, try to remember the identity on your own before checking your notes. All problems should be attempted without a calculator.

1. Let  $f(x) = \cos\left(x + \frac{\pi}{3}\right)$  and  $g(x) = \sin\left(x - \frac{\pi}{6}\right)$ . In the  $xy$ -plane, what are the  $x$ -coordinates of the points of intersection of the graphs of  $f$  and  $g$  for  $0 \leq x < 2\pi$ ?
2. Let  $f(x) = \sin(2x) + 2 \cos x$  and  $g(x) = \sin x + 1$ . In the  $xy$ -plane, what are the  $x$ -coordinates of the points of intersection of the graphs of  $f$  and  $g$  for  $0 \leq x < 2\pi$ ? Hint: factor by grouping.
3. Consider the sinusoidal functions  $f$  and  $g$ , defined by  $f(x) = \cos\left(x - \frac{\pi}{4}\right)$  and  $g(x) = \sin\left(x + \frac{7\pi}{4}\right)$ . Find all zeros of the function  $h$  defined by  $h(x) = f(x) + g(x)$  on the interval  $[0, 2\pi]$ .
4. Find all values of  $\theta$  for which
$$\frac{\sin\left(\theta + \frac{5\pi}{6}\right) + \cos\left(\theta + \frac{2\pi}{3}\right)}{\cos\left(\theta - \frac{\pi}{3}\right) - \sin\left(\theta - \frac{\pi}{6}\right)} = 3$$
5. The function  $h$  is given by  $h(\theta) = \frac{\sin(2\theta)}{\sin\left(\frac{\pi}{2} - \theta\right)}$ . Find all input values in the domain of  $h$  that yield an output value of  $-1$ .

6. The function  $g$  is given by  $g(\theta) = \cos(2\theta) + \cos(\theta) - 2\sin^2(\theta) + 3$

a. Rewrite  $g$  as function of  $\cos(\theta)$  only.

b. Which of the following are zeros of  $g$ ? Select all.

|                                   |                                         |                                         |                                          |
|-----------------------------------|-----------------------------------------|-----------------------------------------|------------------------------------------|
| $0$                               | $\frac{\pi}{2}$                         | $\pi$                                   | $\frac{3\pi}{2}$                         |
| $\arccos\left(\frac{1}{4}\right)$ | $\pi - \arccos\left(\frac{1}{4}\right)$ | $\pi + \arccos\left(\frac{1}{4}\right)$ | $2\pi - \arccos\left(\frac{1}{4}\right)$ |

7. The function  $m$  is given by

$$m(x) = \arccos(3x + 1).$$

Find all input values in the domain of  $m$  that yield an output value of  $\frac{2\pi}{3}$ .

8. The function  $f$  is given by  $f(\theta) = \arcsin\left(\frac{\tan \theta}{2}\right)$ . Solve  $f(\theta) = \frac{\pi}{6}$  for all values of  $\theta$  in the domain of  $f$ .

9. The function  $g$  is given by  $g(x) = 2 \arctan(x + 1)$ . Solve  $g(x) = \arcsin\left(\frac{\sqrt{3}}{2}\right)$  for all values of  $x$  in the domain of  $g$ .

Apply trigonometric identities to match each expression on the left with the numerically equivalent expression on the right. Use each expression exactly once.

10.  $4 \sin\left(\frac{5\pi}{12}\right) \cos\left(\frac{3\pi}{4}\right) + 4 \cos\left(\frac{5\pi}{12}\right) \sin\left(\frac{3\pi}{4}\right)$

11.  $\left(\sin\frac{\pi}{8} + \cos\frac{\pi}{8}\right)^2$

12.  $\cos\left(\frac{7\pi}{5}\right) \cos\left(\frac{2\pi}{5}\right) + \sin\left(\frac{7\pi}{5}\right) \sin\left(\frac{2\pi}{5}\right)$

13.  $\frac{1 - \tan^2\left(\frac{\pi}{8}\right)}{\sec^2\left(\frac{\pi}{8}\right)}$

a.  $\frac{1}{\sqrt{2}}$

b.  $1 + \frac{1}{\sqrt{2}}$

c.  $-1$

d.  $-2$

## Solving Trig Equations Using Advanced Identities HW Answers

1.  $x = \frac{\pi}{6}$  and  $x = \frac{7\pi}{6}$
2.  $x = \frac{\pi}{3}, x = \frac{3\pi}{2}, x = \frac{5\pi}{3}$
3.  $0, \pi,$  and  $2\pi$
4.  $\theta = \frac{2\pi}{3} + \pi k$ , where  $k$  is any integer.
5.  $\theta = \frac{7\pi}{6} + 2\pi k$  and  $\theta = \frac{11\pi}{6} + 2\pi k$ , where  $k$  is any integer.
6.
  - a.  $g(\theta) = 4 \cos^2 \theta + \cos \theta$
  - b.  $\frac{\pi}{2}, \pi - \arccos\left(\frac{1}{4}\right), \pi + \arccos\left(\frac{1}{4}\right), \frac{3\pi}{2}$
7.  $x = -\frac{1}{2}$
8.  $\theta = \frac{\pi}{4} + \pi k$ , where  $k$  is any integer
9.  $x = \frac{1}{\sqrt{3}} - 1$
10. d.  $-2$
11. b.  $1 + \frac{1}{\sqrt{2}}$
12. c.  $-1$
13. a.  $\frac{1}{\sqrt{2}}$